Comparison between finite difference and finite element methods: application to biological cell exposure

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Abstract — The finite element and finite difference methods are compared for the modelling of a biological cell exposed to an electric pulse.

I. Introduction

In this work, the performances and the ease of implementation of the Finite Difference (FD) and the Finite Element (FE) methods are compared for the modelling of biological cell exposed to electric fields. The electro-quasistatic formulation (1) is considered in both time-harmonic and time-transient versions. The model of the cell is the simplest: a conductive cytoplasm surrounded by an insulating membrane [1]. This model is 3D axi-symmetric. The comparison is performed in terms of the computed transmembrane potential (TMP), which is defined as the difference between the outside and inside faces of the membrane.

$$-\mathrm{div}\left(\left(\sigma+\varepsilon\frac{\partial}{\partial t}\right)\mathrm{grad}\left(V\right)\right)=0 \qquad (1)$$

II. Time-harmonic comparison

In FE, an approximate transmission condition is implemented in order to avoid meshing the thin membrane (5 nm of thickness) [2]. In FD, the mesh is non-conforming to the membrane; a uniform grid is used with a five-point stencil whose coefficients are evaluated by harmonic averaging [3].

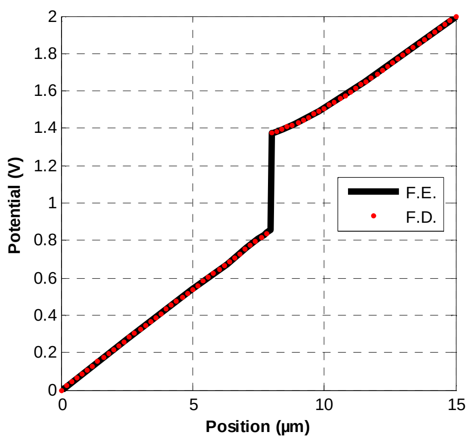

Fig. 1 Potential along one symmetry axis of the cell at 5 MHz computed with both methods.

Fig. 2 shows the electric potential along one symmetry axis (from the center of the cell to the upper electrode). The potential "jump" characterizes the TMP. The relative difference between results obtained with both FE and DF methods is less than 1%. Compared to analytical values [4], the error is below 4%.

III. Time-transient comparison

The time discretization is based on the Crank Nicholson scheme in the FE case and on the implicit Euler scheme in the FD case.

The top of Fig. 2 depicts the 300 nanosecond pulse applied to the cell. The bottom shows the TMP computed with both FD and FE methods. In FD, results are obtained with different grid steps (from 200 nanometers to 50 nm). When considering the finest FD grid, both methods are in good agreement: the maximal difference is below 1%. From Table I, it is shown that a fine mesh is required in our implementation of the FD method.

TABLE I
RESULTS OF THE COMPARISON

	FE	FD 50nm	FD 100nm	FD 200nm
Unknowns	15 000	160 000	40 000	10 000
Error	reference	0.8%	3.5%	15%

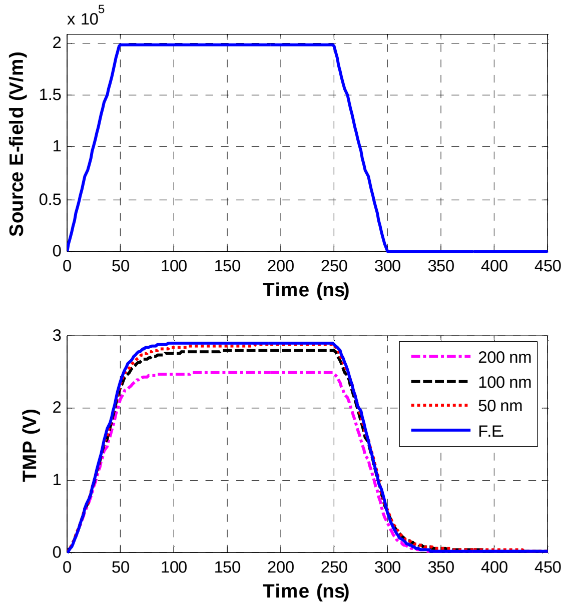

Fig. 2 Time-transient response of the cell (Max. source E-field: 2.10^5 V/m).

IV. Conclusion

If electro-quasistatic formulation is sufficient to model the phenomena, the FE method gives the best result in terms of the ratio between accuracy and numerical costs (Table I). In this application, the solution of the full Maxwell system is not relevant with the FD method. Because of the mesh size, the Courant-Friedrichs-Levy stability condition imposes a time step which is too small compared to the duration of the pulse.

V. References

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